Two planes on the same route are:
--separated
--trailing plane is traveling faster than the leading plane

SMART SKIES™ FLYBY MATH™

Distance-Rate-Time Problems in Air Traffic Control

for Grades 5 - 9

AIR TRAFFIC CONTROL PROBLEM 5

Teacher Guide with Answer Sheets

Overview of Air Traffic Control Problem 5

In this Air Traffic Control (ATC) Problem, students will determine when two airplanes traveling on the same route will conflict with (meet) one another. This will determine when the controller will have to take action to keep the planes separated.

Initially, the airplanes are each a **different distance from the start of the jet route**.
Each airplane is traveling at a **different constant (fixed) speed**, and the trailing airplane is traveling faster than the leading plane (the plane with a "headstart").

This scenario is the first to address two planes traveling on the same route, so it differs from the previous four problems which each involve merging aircraft. The current problem is closest in concept to ATC Problem 4 (the only other "different speeds" scenario) in which two planes are also traveling at different speeds and one plane also has a "headstart."

Objectives

Students will determine the following:

- If two planes are traveling at different speeds on the same route and the trailing plane is traveling faster than the leading plane, the trailing plane will close the gap at a rate equal to the difference in the speeds of the planes.

- So if the difference in speeds is twice as great and the starting distance between the planes remains the same as the original starting separation distance, then the trailing plane will close the gap at twice the original rate. Therefore, the amount of time for the trailing plane to catch up to the leading plane will be half as great.

- If the planes each travel at their original speeds but the starting distance between the planes is twice as great, then the trailing plane must now close twice the distance. However, the plane is traveling at the original rate. So the amount of time will double for the trailing plane to catch up to the leading plane.

Materials

Student handouts:

ATC Problem 5 Student Workbook
ATC Problem 5 Assessment Package (optional)

The student handouts are available on the *FlyBy Math*™ website:

http://www.smartskies.nasa.gov/flyby

Materials for the experiment:

--sidewalk chalk or masking tape or cashier's tape or a knotted rope
--measuring tape or ruler
--marking pens (optional)
--1 stopwatch or 1 watch with a sweep second hand or
1 digital watch that indicates seconds
--pencils
--signs (available on the *FlyBy Math*™ website) identifying pilots, controllers, and NASA scientists
--clipboard (optional)

Introducing Your Students to the ATC Problem

You may want to show the FlyBy Math™ video clips to introduce your students to the air traffic control system.

(For more detail, see the FlyBy Math™ Educator Guide.)

To help your students understand the problem, you can ask them to consider this related problem that is set in a more familiar context:

Two students, Ana and Alex, live on the same street. They each leave their homes at the same time and walk in the same direction along their street at different constant (fixed) speeds. Alex starts out 10 blocks behind Ana, but Alex walks twice as fast as Ana. Both students are walking to a school at the end of their street, 30 blocks beyond Ana's house.

You can ask your students this question:

Will Alex catch up with Ana before she reaches the school?

Student Workbook

For a detailed description of the Student Workbook features found in each ATC Problem, see the *FlyBy Math*™ Educator Guide.

The following section addresses *special features* of the ATC Problem 5 Workbook.

It may help your students to think of the plane speeds in inches per second:
6 inches/second
3 inches/second

Read the Problem
The speed of one airplane is 1/2 foot/second. The speed of the other airplane is 1/4 foot/second. The faster plane starts 10 feet behind the slower plane.

Note: The speeds and distance were chosen to reflect the classroom experiment that the students will conduct and are not related to real-world parameters.

As a problem enhancement, you may want to ask your students to solve the problem using real-world data.

In a real-world scenario, one plane might be traveling at 400 nautical miles per hour and the other plane might be traveling at 320 nautical miles per hour. The initial separation distance might be 10 nautical miles.

An international nautical mile is 1,852 meters.

A nautical mile per hour is called a "knot".

Set Up and Do the Experiment
A complete description of this section is contained in the *FlyBy Math™* Educator Guide.

If both pilots walk one behind the other on the jet route, it will be difficult for the NASA Scientists and the Air Traffic Controller to track the pilots' progress. To make it easier, have one pilot walk down the left side of the jet route and the other pilot walk down the right side. To accomplish this, place the speed control lines for each plane on opposite sides of the jet route. Make the marks or pieces of tape at least 1 foot long.

Do the Calculations
Each of the six calculation methods is described in the *FlyBy Math™* Educator Guide.

Notice that the jet route diagrams feature six copies of the jet route. These copies represent "snapshots" of the jet route at 10-second intervals. As students plot points on each copy, they will see the trailing plane close in on the leading plane.

Note that if the horizontal "tick marks" on each jet route were extended and connected, a grid would be formed. This grid would be equivalent to the grid given in the calculation method "Plot Points on a Grid."

Three calculation methods are described in greater detail below.

--Count Feet and Seconds
In the previous ATC Problems, students tracked the total number of seconds each plane traveled to reach the point where the routes merged. In the current ATC Problem, students look at the data for both planes to see when the planes will be in the same place at the same time.

--Use a Formula
In the previous ATC Problems, both the distance to the intersection and the rate were known for each airplane. So each plane's travel time to the intersection could be calculated by simply substituting its distance and rate in the Distance-Rate-Time formula written as $t = d/r$.

In the current ATC Problem, only the rate is known for each plane. So the formula t = d/r does not lead to a direct answer. Instead, the formula d = rt is used to obtain an equation for each plane that describes the plane's distance traveled as a function of time traveled. In this ATC Problem, the "headstart" is quite easy to picture and it plays a role in the equation for the corresponding plane.

Students use the equations to fill in a table and determine the number of seconds it will take the trailing plane to catch up to the leading plane.

--Graph Two Linear Equations

Caution: Students may confuse the path of a plane with the graph of the plane's distance traveled as a function of time. In the current ATC Problem, the point at which the two lines cross represents the collision of the planes. At that point, the planes are in the same place at the same time.

You may want your students to compare this graph with the corresponding graph in ATC Problem 4. In Problem 4, the planes were on separate routes and the point where the lines crossed merely indicated that at that time, each plane had traveled the same distance from the beginning of its route.

Analyze Your Results

As part of the Analysis, you may also want to ask your students to create a similar problem in a different setting. They have already considered a problem in which two students walk from their respective homes to their school. (See the *Introducing Your Students to the ATC Problem* section of this document.)

Now, you might suggest they consider two cars traveling in the same lane on the same road. Each car is traveling at a different constant (fixed) speed. The trailing car is traveling faster than the leading car.

Note: To be consistent with the airspace scenarios, it is important that for each problem created by you or your students, you choose a fixed (constant) speed for each vehicle or person. (For example, a rocket launch scenario would *not* be appropriate because a launched rocket typically accelerates and therefore its speed is not constant.)

Extension

The extension introduces a separation requirement along the jet route. For safety reasons, the planes always must be separated by a distance greater than or equal to a given standard separation distance. If their separation is less than this standard, a separation violation will occur.

Students are asked to review their calculations to determine when the planes first violate the separation requirement.

Answers and Explanations

The first part of this section summarizes the answers to the key questions posed in the ATC Problem. The remainder of this section is organized by activity and includes graphs, diagrams, and answers to individual questions, as well as discussions of the general problems posed in the analysis activity and the posttest.

Answer Summary

The speed of Flight WAL27 is 1/2 foot/second, so the plane travels 1/2 foot in 1 second.
The speed of Flight NAL63 is 1/4 foot/second, so the plane travels 1/4 foot in 1 second.
The planes are traveling on the same route.
At the start of the problem, the faster plane (Flight WAL27) is 10 feet behind the slower plane (Flight NAL63). That is, Flight NAL63 has a 10-foot "headstart."

Since the trailing plane is traveling more rapidly that the leading plane, the trailing plane will catch up to the leading plane.

In particular:

- It will take 40 seconds for Flight WAL27 to close the 10-foot gap and catch up to Flight NAL63.

- The speed of Flight WAL27 is 1/4 foot per second greater than the speed of Flight NAL63.

- The trailing plane closes the gap at a rate equal to the difference in the speeds of the two planes. So Flight WAL27 closes the 10-foot gap at a rate of 1/4 foot per second, the difference in the plane speeds.
- If the speed of Flight WAL27 were increased to 3/4 foot per second, then the difference in the speeds of the two planes would be 1/2 foot per second. (That is double the difference—1/4 feet per second—of the original speeds.) Then Flight WAL27 would close the 10-foot gap at a rate of 1/2 foot per second. (That is double the original closing rate.) It will take 20 seconds to close the gap.

Answers by Activity

Answers are provided for all worksheets including the Pretest, Student Workbook, and Posttest. Please see the following pages.

Note: Answers are given only for the numbered activity steps that require students to provide a numerical or verbal response. For example, if a step requires a student to trace or circle a portion of a diagram, that step is *not* included in the Answers.

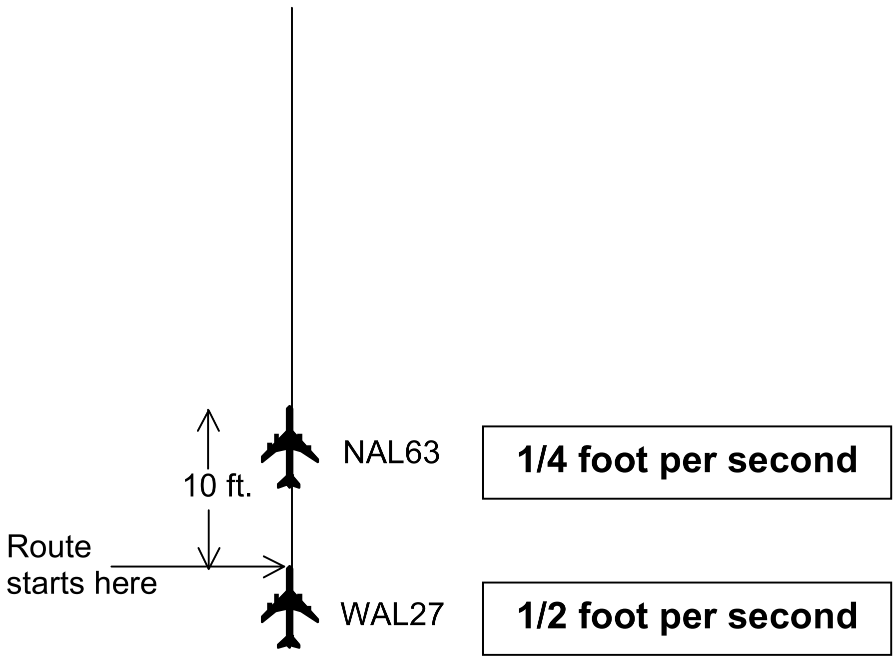
ANSWERS BY ACTIVITY

Pretest

1. How many seconds will it take Flight WAL27 to close the 10-foot gap and catch up with NAL63? **40 seconds**

   Explain your reasoning. **Flight WAL27 closes the gap at a rate equal to the difference in the speeds of the two planes. So Flight WAL27 closes the gap at a rate of 1/4 foot per second. At that rate, it will take 40 seconds for Flight WAL27 to close the 10-foot gap.**

Student Workbook: Read the Problem



2. How far does NAL63 travel in 1 second? **1/4 foot**

3. How far does NAL63 travel in 10 seconds? **$2\frac{1}{2}$ feet**

5. How far does WAL27 travel in 1 second? **1/2 foot**

6. How far does WAL27 travel in 10 seconds? **5 feet**

Student Workbook: Do the Calculations—Count Feet and Seconds

Discussion: Flight NAL63 travels 1 foot in 4 seconds. Count by 4s to fill in the seconds in the blanks along the jet route.
Flight WAL27 travels 1 foot in 2 seconds. Count by 2s to fill in the seconds in the blanks along the jet route.
Flight NAL63 has a 10-foot "headstart."

At 40 seconds, Flight WAL27 and Flight NAL63 will each be 20 feet from the point where the route begins. So in 40 seconds, Flight WAL27 will close the 10-foot gap and catch up with Flight NAL63.

23 ft.	**52** sec.	**46** sec.	23 ft.
22	**48** sec.	**44** sec.	22
21	**44** sec.	**42** sec.	21
20	**40** sec.	**40** sec.	20
19	**36** sec.	**38** sec.	19
18	**32** sec.	**36** sec.	18
17	**28** sec.	**34** sec.	17
16	**24** sec.	**32** sec.	16
15	**20** sec.	**30** sec.	15
14	**16** sec.	**28** sec.	14
13	**12** sec.	**26** sec.	13
12	**8** sec.	**24** sec.	12
11	**4** sec.	**22** sec.	11
10	10-foot headstart	**20** sec.	10
9		**18** sec.	9
8	NAL63	**16** sec.	8
7		**14** sec.	7
6		**12** sec.	6
5		**10** sec.	5
4		**8** sec.	4
3		**6** sec.	3
2		**4** sec.	2
1		**2** sec.	1
0			0
feet		WAL27	**feet**

Student Workbook: Do the Calculations—Count Feet and Seconds (cont.)

7. How many seconds will it take WAL27 to close the 10-foot gap and catch up with NAL63? That is, after how many seconds will WAL27 and NAL63 be in the same place at the same time? __**40**__ seconds

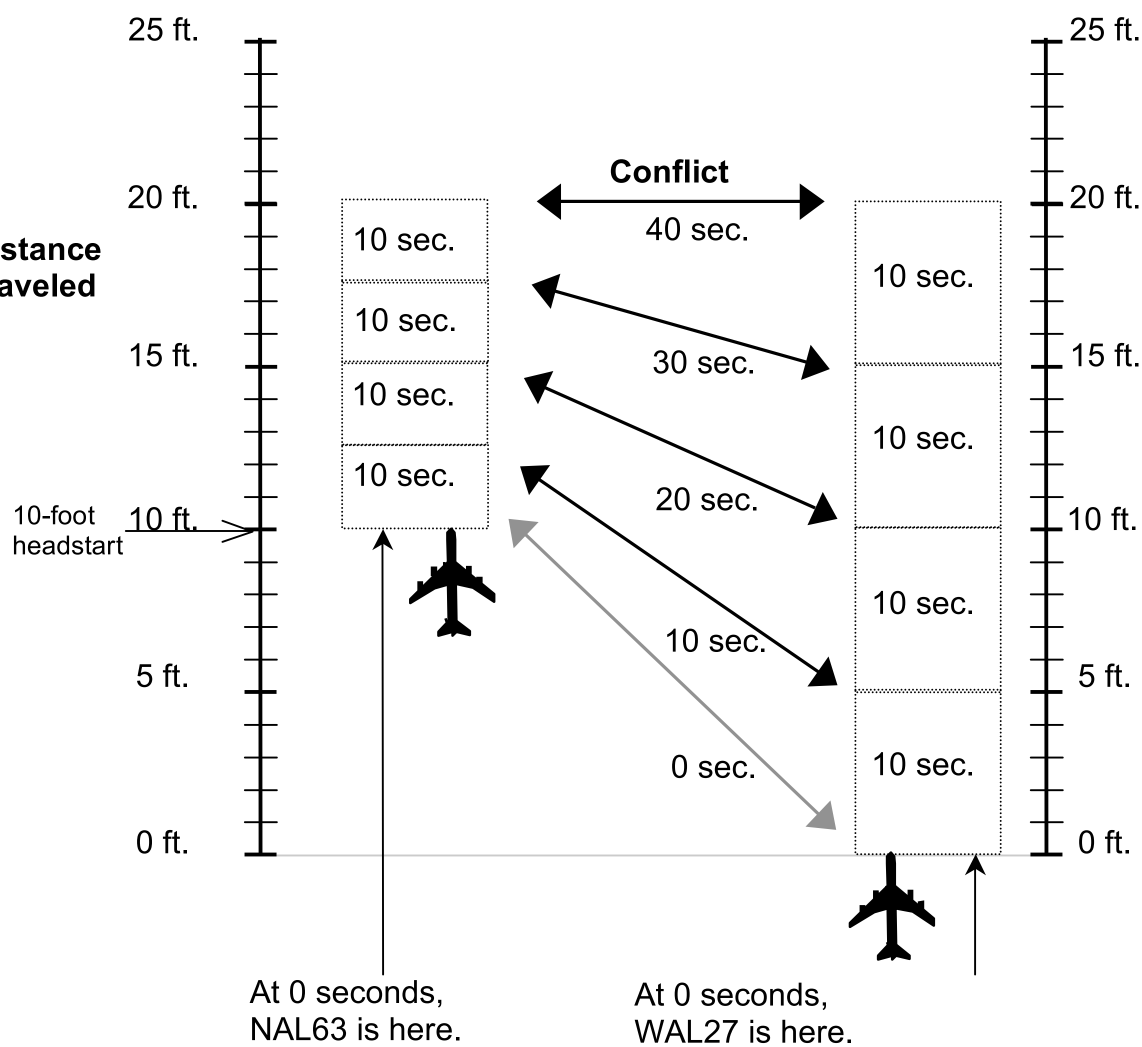
8. What would you tell the air traffic controller to do to avoid a collision?

   Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Draw Blocks

Discussion:
Flight NAL63 travels 1 foot in 4 seconds. So in 10 seconds, Flight NAL63 will go $2^1/_2$ feet.
Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.
The following diagram shows a stack of 10-second blocks for each plane.
In the stack corresponding to Flight NAL63, each block represents $2^1/_2$ feet.
In the stack corresponding to Flight WAL27, each block represents 5 feet.

By stacking a block for Flight NAL63 and immediately stacking the corresponding block for Flight WAL27, it will become clear that each stack requires 4 blocks to reach the same height. When the stacks are the same height, a conflict will occur. This happens at 40 seconds, when each plane is 20 feet from the starting position of Flight WAL27.



Student Workbook: Do the Calculations—Draw Blocks (cont.)

16. How many seconds will it take WAL27 to close the 10-foot gap and catch up with NAL63? That is, after how many seconds will WAL27 and NAL63 be in the same place at the same time? **40** seconds

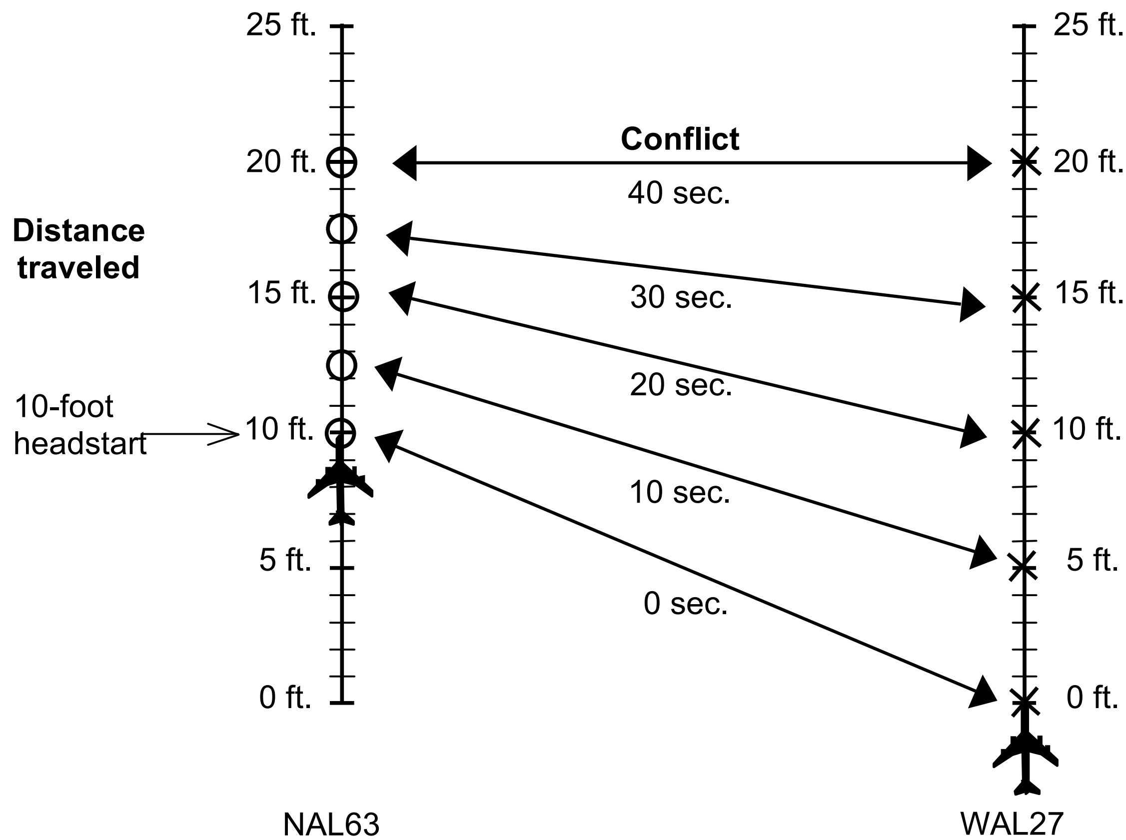
17. What would you tell the air traffic controller to do to avoid a collision?

    Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Plot Points on Lines

Discussion:
Flight NAL63 travels 1 foot in 4 seconds. So in 10 seconds, Flight NAL63 will go $2^1/_2$ feet.
Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.
The following diagram shows the position of each plane at 10-second intervals.

By plotting a point for Flight NAL63 and immediately plotting the corresponding point for Flight WAL27, students will see that when the points are the same distance from the start of the jet route, a conflict will occur. This happens at 40 seconds, when each plane is 20 feet from the starting position of Flight WAL27.



13. How many seconds will it take WAL27 to close the 10-foot gap and catch up with NAL63? That is, after how many seconds will WAL27 and NAL63 be in the same place at the same time? **40** seconds

Student Workbook: Do the Calculations—Plot Points on Lines (cont.)

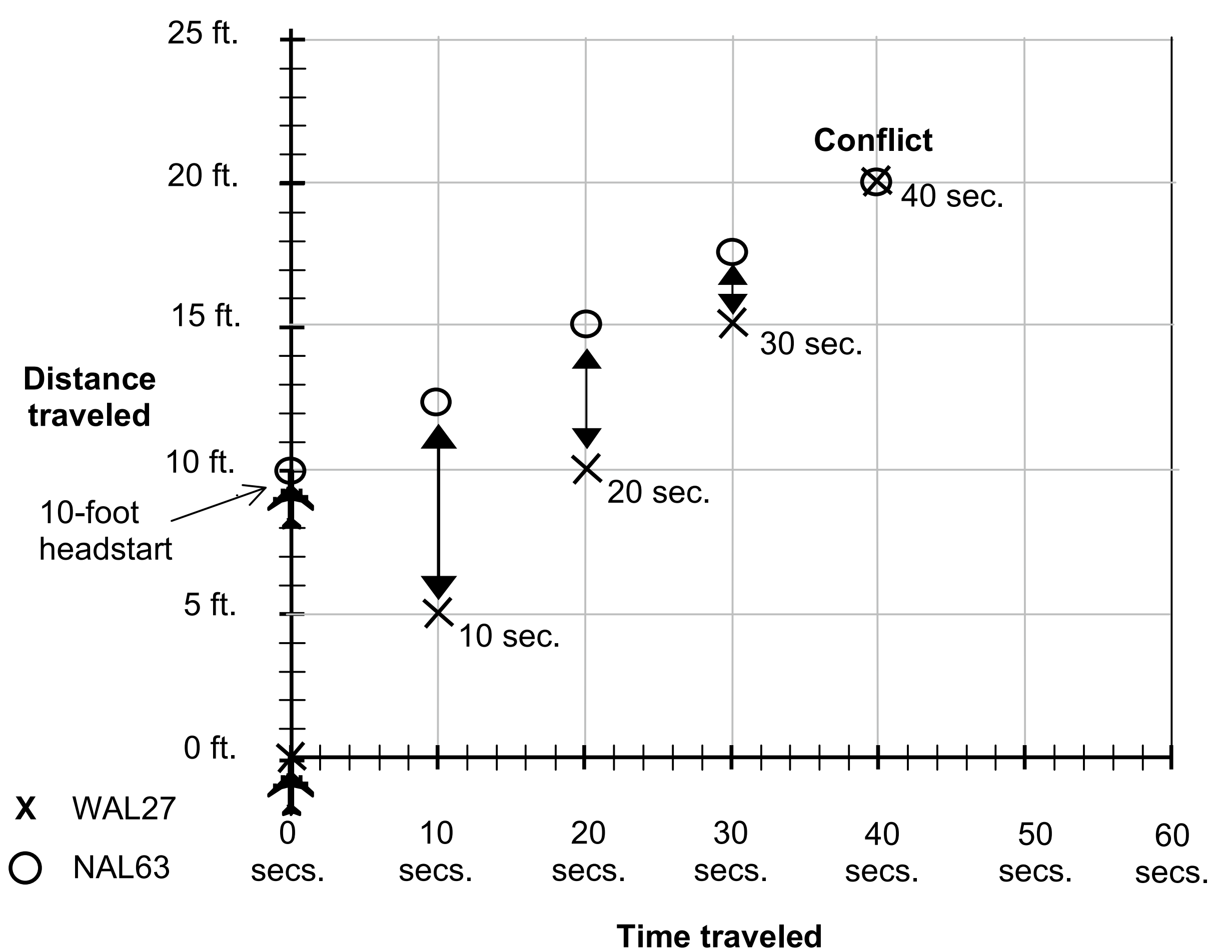
14. What would you tell the air traffic controller to do to avoid a collision?

Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Plot Points on a Grid

Discussion:
Flight NAL63 travels 1 foot in 4 seconds. So in 10 seconds, Flight NAL63 will go $2^{1}/_{2}$ feet.
Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.
The following diagram shows the position of each plane at 10-second intervals.

By plotting a point for Flight NAL63 and immediately plotting the corresponding point for Flight WAL27, students will see that when the points are the same distance from the start of the jet route, a conflict will occur. This happens at 40 seconds, when each plane is 20 feet from the starting position of Flight WAL27.



Student Workbook: Do the Calculations—Plot Points on a Grid (cont.)

12. How many seconds will it take WAL27 to close the 10-foot gap and catch up with NAL63? That is, after how many seconds will WAL27 and NAL63 be in the same place at the same time? **40** seconds

13. What would you tell the air traffic controller to do to avoid a collision?

    Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Use a Formula

Discussion: When t = 40 seconds, each plane has traveled 20 feet from the point where the route begins. So a conflict will occur at t = 40 seconds.

1. -In 4 seconds, Flight WAL27 travels **0.5 feet/second** × **4 seconds** = **2.0** feet.

   -In 5 seconds, Flight WAL27 travels **0.5 feet/second** × **5 seconds** = **2.5** feet.

2. How could you use multiplication to find the distance Flight WAL27 travels in 14 seconds? **Multiply 0.5 feet/second by 14 seconds.**

3. Use the formula

   d = r • t

   to answer this question.

   How many feet does Flight WAL27 travel in 20 seconds? **10 feet**

4. Use the formula

   d = r • t

   to answer this question.

   How many feet does Flight NAL63 travel in 20 seconds? **5 feet**

Student Workbook: Do the Calculations—Use a Formula (cont.)

NAL 63	WAL27
d = (1/4) t + 10	d = (1/2) t

5. For each plane, use its equation to fill in the table.

	0 sec.	10 sec.	20 sec.	30 sec.	40 sec.
Distance traveled by Flight NAL63	**10 ft.**	**12.5 ft.**	**15 ft.**	**17.5 ft.**	**20 ft.**
Distance traveled by Flight WAL27	**0 ft.**	**5 ft.**	**10 ft.**	**15 ft.**	**20 ft.**

6. How many seconds will it take WAL27 to close the 10-foot gap and catch up with NAL63? That is, after how many seconds will WAL27 and NAL63 be in the same place at the same time? **40** seconds

7. What would you tell the air traffic controller to do to avoid a collision?

Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Graph Linear Equations

Discussion: The planes will conflict at 40 seconds, the time that corresponds to the point where the two lines intersect. At 40 seconds, each plane is 20 feet from the beginning of the jet route. Since the planes are traveling on the same route, the planes intersect at the point where their graphs intersect.

1. Fill in the table for **NAL63**.

d = (1/4)t + 10

t seconds	**d** feet
0	**10**
10	**12.5**
20	**15**
30	**17.5**
40	**20**

2. Fill in the table for **WAL27**.

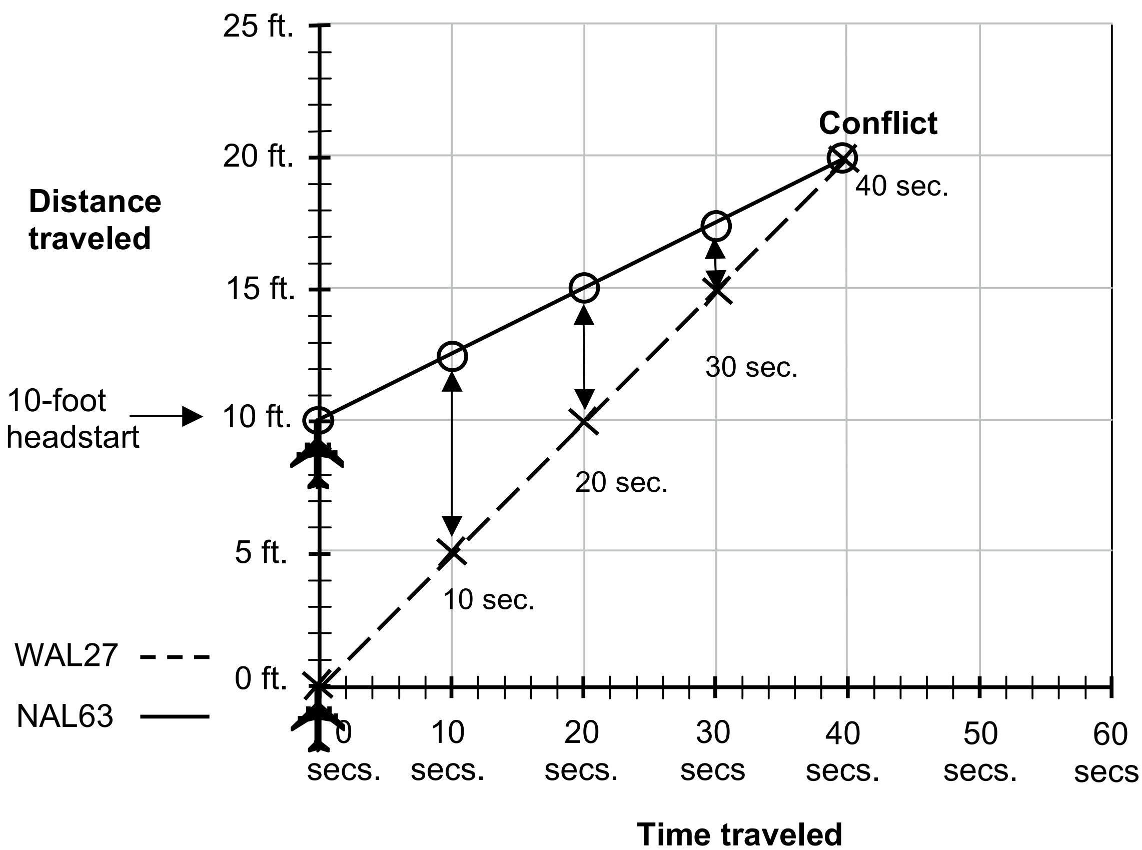
d = (1/2)t

t seconds	**d** feet
0	**0**
10	**5**
20	**10**
30	**15**
40	**20**

3. Use an **O** to graph each point in the NAL63 table. Use a solid line to connect the points.

4. Use an **X** to graph each point in the WAL27 table. Use a dotted line to connect the points.

Student Workbook: Do the Calculations—Graph Linear Equations (cont.)



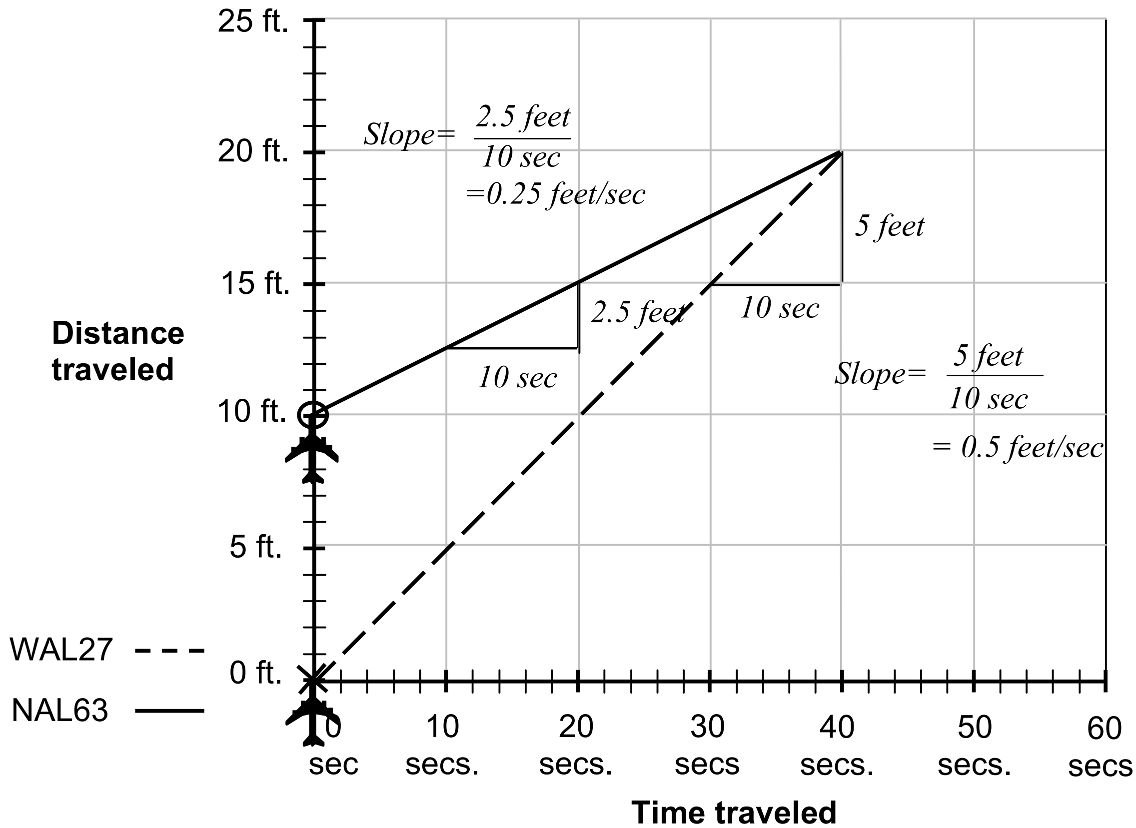
5. How many seconds will it take WAL27 to close the 10-foot gap and catch up with NAL63? That is, after how many seconds will WAL27 and NAL63 be in the same place at the same time? **40** seconds

6. What would you tell the air traffic controller to do to avoid a collision?

Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Graph Linear Equations (cont.)

Discussion: For each line, its slope represents the speed of the corresponding plane.
For the line corresponding to Flight WAL27, the slope of the line is 0.5 feet/sec, the speed of Flight WAL27.
For the line corresponding to Flight NAL63, the slope of the line is 0.25 feet/sec, the speed of Flight NAL63.



7. Write the number that is the slope of the line representing NAL63. **<u>1/4 feet/second</u>**

8. Write the number that is the slope of the line representing WAL27. **<u>1/2 feet/second</u>**

9. What information does the slope of the line tell you about each plane?

For Flight NAL63, the slope of the line is 1/4 foot/second, the speed of Flight NAL63. For Flight WAL27, the slope of the line is 1/2 foot/second, the speed of Flight WAL27.

Student Workbook: Analyze Your Results

- The WAL27 speed is 1/2 foot per second.
- The NAL27 speed is 1/4 foot per second.

5. What is the difference in speed between WAL27 and NAL63?
   That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? **1/4 foot per second**

6. How fast is WAL27 closing the gap between the planes?
   That is, the distance between the planes changes how many feet per second? **1/4 foot per second**

7. What is the relationship between the difference in speeds and the speed at which WAL27 is closing the gap? **They are the same.**
   WAL27 is closing the gap at a rate equal to the difference in the speeds of the planes.

- Suppose the speed of WAL27 were **3/4** foot per second.

8. With this new faster speed, at how many feet per second would WAL27 close the gap? **1/2 foot per second**

9. With the new faster speed, how many seconds will it take to close the gap? **20 seconds**

Student Workbook: Analyze Your Results (cont.)

Two planes are flying at different speeds on the same route. The planes start at different distances from the beginning of the route.

10. Do you have enough information to predict whether the trailing plane will catch up to the leading plane? **No**

11. If NO, what other information do you need? **The plane speeds and the distance between the planes; whether or not the trailing plane is traveling faster than the leading plane.**

- Now suppose the trailing plane is traveling faster than the leading plane.
- Also suppose the difference in speeds is twice as great.

12. What would you expect to happen to the amount of time it would take the trailing plane to catch up to the leading plane? **The amount of time for the trailing plane to catch up to the leading plane would be half as great.**

    Why? **When the difference in speeds is twice as great, the trailing plane is closing the gap at twice the original rate. So the gap will be closed in half as much time.**

Student Workbook: Analyze Your Results (cont.)

- Suppose the planes each travel at their original speeds, but the distance between the planes is twice as great.

13. What would you expect to happen to the amount of time it would take the trailing plane to catch up to the leading plane? **The amount of time for the trailing plane to catch up to the leading plane would be twice as great.**

    Why? **When the gap is twice as great, the trailing plane must close twice the distance at the original rate. So it will take twice as much time to close the gap.**

Student Workbook: Extension

1. How many seconds will it take WAL27 to close the gap from 10 feet to 5 feet? **20 seconds**

2. What would you tell the air traffic controller to do to be sure the planes always meet the separation requirement?

Change the speed or change the route of one of the planes.

Posttest

1. How many seconds will it take Flight WAL27 to close the gap from 10 feet to 5 feet? That is, after how many seconds will a separation violation occur?

   15 seconds

2. What is the difference in speed between Flight WAL27 and Flight NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane? **1/3 foot per second**

3. How fast is Flight WAL27 closing the gap between the planes? That is, the distance between the planes changes how many feet each second? **1/3 foot per second**

4. What is the relationship between the difference in speeds and the speed at which Flight WAL27 is closing the gap? **They are the same.**

5. The speed of Flight WAL27 is **1** foot per second. Now suppose the speed of Flight WAL27 were **1** $^{1}/_{3}$ feet per second. With this new faster speed, at how many feet per second would Flight WAL27 close the gap? With the new faster speed, how many seconds will it take to close the gap from 10 feet to 5 feet?

   The new difference in speeds is 2/3 feet per second. This is twice as great as the original difference in speeds, so WAL27 will close the gap in half the time. That is, WAL27 will close the gap from 10 feet to 5 feet in 7.5 seconds.

Posttest (cont.)

6. What would you tell the air traffic controllers to do to be sure the planes always meet the separation standard?

Change the speed or change the route of one of the planes.

Now consider this general problem.

Two planes are traveling at different speeds on the same route.

The trailing plane is traveling faster than the leading plane.

7. Do you have enough information to predict how long it will take the trailing plane to close one-half of the starting gap between the planes? Why or why not?

No. You need to know the plane speeds and the starting separation distance.

8. Now suppose the difference in speeds is twice as great.
   What would you expect to happen to the amount of time it would take the trailing plane to close one-half of the starting gap between the planes?

The amount of time for the trailing plane to catch up to the leading plane would be half as great.

Why? **When the difference in speeds is twice as great, the trailing plane is closing the starting gap at twice the original rate. So one-half of the starting gap will also be closed in half as much time as with the original speeds.**

Posttest (cont.)

9. Finally, suppose that the planes each travel at their original speeds, but the distance between the planes is twice as great.

   What would you expect to happen to the amount of time it would take the trailing plane to close one-half of the *new* starting gap between the planes?

   The amount of time for the trailing plane to catch up to the leading plane would be twice as great.

   Why? **When the gap is twice as great, the trailing plane must close twice the distance at the original rate. So it will also take twice as much time to close one-half of the new starting gap as it did with the original distance.**